A British decision

Cecina Babich Morrow

March 2026

1 Problem set-up

State of nature: Will it rain tomorrow?
Weather forecast: **67%** chance of rain tomorrow – this is our *prior probability*

Decision options:

1. Wear a raincoat
2. Don't wear a raincoat

Utility: What is the utility of each of our decisions, given the weather tomorrow (the true state of nature)?
What are some factors we might prioritise as decision-makers?

Decision / Weather	Rain tomorrow	No rain tomorrow
Wear a raincoat		
Don't wear a raincoat		

Table 1: Utilities for each decision and possible state of nature.

2 Make your initial decision

What decision do you want to make?

☐ Wear a raincoat

☐ Don't wear a raincoat

3 How does your decision work out for you?

Now it is tomorrow – let's find out how your decision paid off!

1. Roll your dice:
   - If you get a 1, 2, 3, or 4, it rained ($P(\text{rain}) = 0.67$)
   - If you get a 5 or 6, it stayed dry ($P(\text{no rain}) = 0.33$)
2. Find your utility, based on the table above:

4 What is the Bayes optimal decision?

Definition 1. ***Bayes decision under utility*** *U: Select the decision d^* such that*

$$d^* = \arg\max_d \sum_{\theta \in \Theta} U(\theta, d) p(\theta) = \arg\max_d \mathbb{E}[U(d)]$$

where θ is a state of nature in the possible space of states of nature Θ, $U(\theta, d)$ is the utility of making decision d when the true state of nature is θ, and $p(\theta)$ is the probability of state of nature θ.

In our case, let's calculate $\mathbb{E}[U(\text{raincoat})]$:

$$\begin{aligned}
\mathbb{E}[U(\text{raincoat})] &= \sum_{\theta \in \Theta} U(\theta, \text{raincoat}) p(\theta) \\
&= U(\text{rain}, \text{raincoat}) \times p(\text{rain}) + U(\text{no rain}, \text{raincoat}) \times p(\text{no rain}) \\
&=
\end{aligned}$$

And now $\mathbb{E}[U(\text{no raincoat})]$:

$$\begin{aligned}
\mathbb{E}[U(\text{no raincoat})] &= \sum_{\theta \in \Theta} U(\theta, \text{no raincoat}) p(\theta) \\
&= U(\text{rain}, \text{no raincoat}) \times p(\text{rain}) + U(\text{no rain}, \text{no raincoat}) \times p(\text{no rain}) \\
&=
\end{aligned}$$

So what is the Bayes optimal decision given our utility from Table 1?

☐ Wear a raincoat

☐ Don't wear a raincoat

5 What if our problem changes?

What is the Bayes optimal decision when our weather app says $p(\text{rain}) = 0.5$? What about if $p(\text{rain}) = 0.3$?

What if we have different priorities as a decision-maker, and thus different utilities?

Decision / Weather	Rain tomorrow	No rain tomorrow
Wear a raincoat		
Don't wear a raincoat		

Table 2: Alternative utilities for each decision and possible state of nature.

What if we have more than two possible decision options? How could we modify our utility table to decide between multiple jackets vs. not wearing one at all?